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Dynamic Modeling of Four – Rotorcraft

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ABSTRACT: In order to achieve the precise control of the four rotorcraft, we must first obtain the accurate mathematical model of the four rotorcraft.This study analyzes the mechanical structure of the four rotorcraft and ignores the effects of the corresponding air resistance and the associated external secondary factors.The actual environment of the four rotorcraft is simplified, and the main factors of the four rotorcraft in motion are seized to establish a more accurate mathematical model.In the modeling process four-rotor aircraft, using the most current hot research field research methods.Mode using the angular velocity and the linear velocity of the separated solver and attitude by the coordinate transformation, and motion Newton Euler's formula to solve.Thereby establishing a nonlinear dynamic model four-rotor aircraft.Finally, a simplified mathematical model of four rotorcraft is obtained by comprehensive analysis of the relevant constraints of mathematical model of four rotorcraft.This makes the accuracy of the model aircraft system higher, more convenient control four rotorcraft.Which has certain reference value and guiding significance for the study of future stability of aircraft system.

Keywords: Four rotorcraft; mechanical structure; mathematical model;

I. INTRODUCTION

An unmanned aerial vehicle (UAV) is an unmanned aerial vehicle that does not require man-made driving and is operated by a radio remote control device or its own program control device[1]. Four rotor unmanned aerial vehicles in the air according to the characteristics of the parameters and control system design to achieve the expected flight path for a series of operations[2,3].Although the mechanical structure of the four rotorcraft is simple, but the four rotorcraft is a strong coupling, underactivity, nonlinear time-varying control system, so the design of the four rotorcraft has some challenges difficult and important research significance.The design and detection and control technology of the four rotorcraft is a multi-disciplinary, multi-domain and multi-crossover integrated technology, which provides a comprehensive research platform for the research of information fusion in many fields such as control theory, advanced sensor technology and computer science and technology[4]. Therefore, this subject has a strong research significance in military, civil and theoretical.For modeling four-rotor aircraft we need to consider the impact of external factors, while the internal parameters is also required. At present, it is of great research value to analyze and study the nonlinear characteristics, controller accuracy and system design of the aircraft, and to reduce the influence of the constraints involved in the design of the aircraft.

II. MECHANICAL STRUCTURE OF FOUR ROTORCRAFT

The object of this paper is the four-rotorcraft with the X-type pricing structure shown in Fig1. Located in the center of the four rotorcraft rack for the four rotorcraft flight control equipment, that is, the controller of the aircraft, referred to as the flight control board[5]. Flight control plate produced by adjusting the variable lift mounted four brushless DC motor at a speed four endpoints quadrotors X type structures to drive the propeller, so that the real-time control of flight attitude four rotorcraft. The four-rotorcraft in practical engineering applications can also add the appropriate protection and application equipment, making the performance of the four rotorcraft more secure.

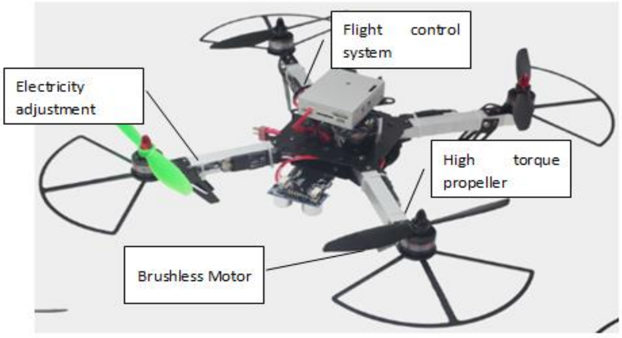

Figure 1 X-type four rotorcraft

III. THE MOTION CHARACTERISTICS OF FOUR ROTORCRAFTS

According to the structural characteristics of the four rotorcraft, it can be seen that the force and torque required for the flight of the four rotorcraft are generated by the four brushless motors driven by the four brushless motors at the end of the four rotorcraft. So the four rotorcraft has four inputs, the four inputs is the end of the four motor voltage. But the four rotorcraft has six degrees of freedom in the direction of the output[6]. The output of these six degrees of freedom is represented by the position and attitude of the four rotorcraft, so the four rotorcraft is a typical underactuated system. The control of the position of the four rotorcraft and the control of the four rotorcraft attitude are inseparable, that is, the control of the position of the four rotorcraft needs to be achieved through the control of the attitude, which fully embodies the strong coupling characteristics of the four rotorcraft.

In order to describe the movement state of the four rotorcraft accurately, this paper makes a further description of the four rotorcraft. Figure 2 shows the motor number and coordinate settings for the four rotorcraft.

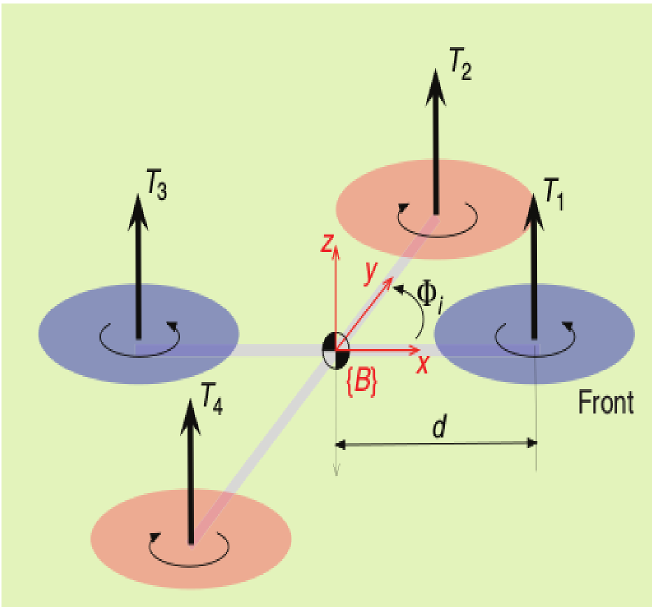

Figure 2 Motor number and coordinate setting of four rotorcraft

The four rotorcraft achieves the change of the movement of the four rotorcraft by changing the speed of the four motors. Reasonable control of the four motor speed can achieve four rotorcraft six degrees of freedom movement. In order to balance the anti-torque generated by the four motors themselves, the four motors are divided into two groups, and the corresponding two motors are grouped together. The two groups of motors are rotated in the opposite direction. As shown in Figure 2, the No. 1 motor and the No. 3 motor are used as a group, in the counterclockwise direction, the No. 2 motor and the No. 4 motor as another group, in accordance with the clockwise rotation[7]. Four rotorcraft can achieve six kinds of control of the movement, respectively, hover and vertical movement, pitching movement, roll movement, yaw movement, front and rear movement, tend to movement. For the convenience of description, this paper establishes the coordinate system for the four rotorcraft, the axis of the No. 1 motor and the motor connected with the No. 3 motor is the X axis and the X axis direction is the direction of the motor No. 3 to No. 1 motor. The shafts connected to the No. 2 motor and the No.

4 motor are Y-axis, and the Y-axis direction is the direction of the motor No. 4 to No. 2 motor. Z axis is perpendicular to the X and Y axis direction plane, Z axis direction up. Coordinate diagram shown in Figure 2.
Four rotorcraft by adjusting the speed of the four rotors, can achieve the following several sports:
(1) hover and vertical movement

Vertical movement is relatively easy. Because the two pairs of motors turn to the opposite, you can balance its anti-torque on the fuselage, while increasing the output power of two pairs of motors, the rotor speed increases the total tension increases, when the total tension is enough to overcome the weight of the machine, The aircraft will rise vertically, and vice versa to reduce the output power of the four motors, the four rotorcraft will drop vertically.

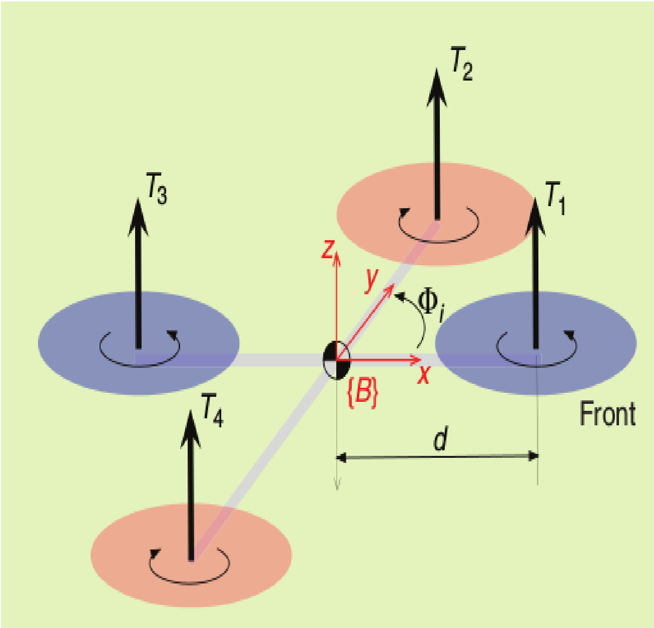

Figure3 Vertical movement

(2) pitching movement

The speed of the motor 1 is increased and the rotational speed of the motor 3 is reduced. The speed of the motor 2 and the motor 4 are kept constant. At this time, the overall torque and the total tension of the aircraft are changed, and the imbalance force is generated to rotate the fuselage. On the contrary, the motor 1 speed down, the motor 3 speed increases, the motor 2 and motor 4 speed unchanged, will be carried over movement.

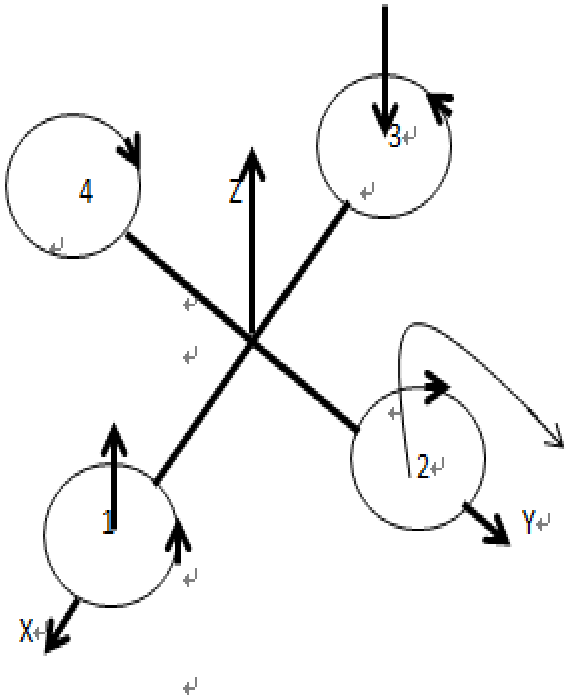

Figure4 pitch movement

(3) roll movement

Change the speed of the motor 2 and the motor 4, keep the motor 1 and motor 3 speed unchanged, you can make the body around the x-axis rotation (forward and reverse), to achieve the roll of the aircraft movement.

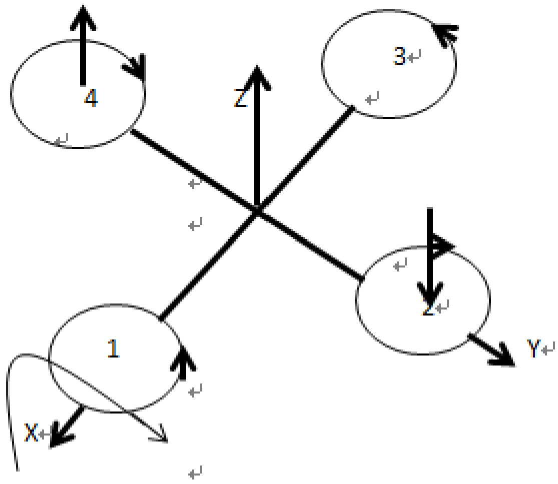

Figure 5 Rolling motion

(4) yaw movement

When the speed of the motor 1 and the motor 3 rises, the torque of the rotor 2 and the motor 4 decreases, the rotor 1 and the rotor 3 are more counter-torque than the rotor 2 and the rotor 4 against the fuselage, Under the action of the torque around the z-axis rotation, to achieve the aircraft yaw movement, steering and motor 1, the reverse direction of the motor 3.

(5) before and after the movement

To achieve the aircraft in the horizontal plane before and after the left and right movement, must face the aircraft in the face of a certain force. Increase the speed of the motor 3, so that the tension increases, the corresponding reduction in the speed of the motor 1 to keep the other two motor speed unchanged, then the aircraft will have a certain degree of tilt, so that the rotor tension to produce horizontal components, The flight of the aircraft before the fly, fly back just the opposite. Of course, the pitch movement will also produce horizontal movement.

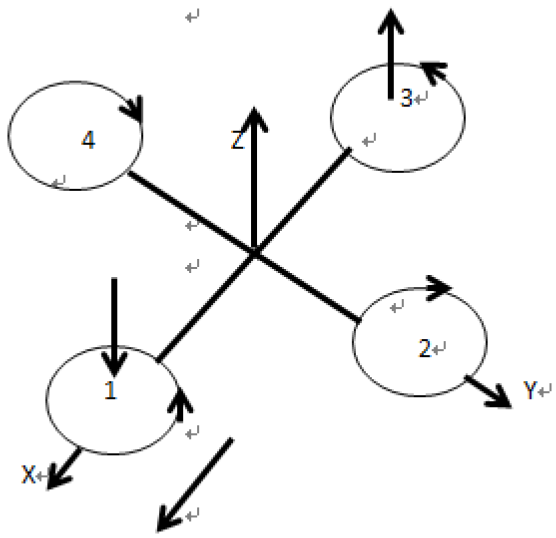

Figure6 before and after the movement

IV. COORDINATE SYSTEM DESCRIPTION AND TRANSFORMATION RELATION

The attitude and position of the four rotorcraft are the core of the four rotorcraft control. For a more accurate study of the motion of the four rotorcraft, we construct a kinetic model for the four rotorcraft, and the four rotorcraft is a 6 degree of freedom rigid The attitude and position of the motion equation and the dynamic equation are used to describe[8]. So, choose a suitable coordinate system is very important. So this paper establishes two coordinate systems. As shown in Figure 7.

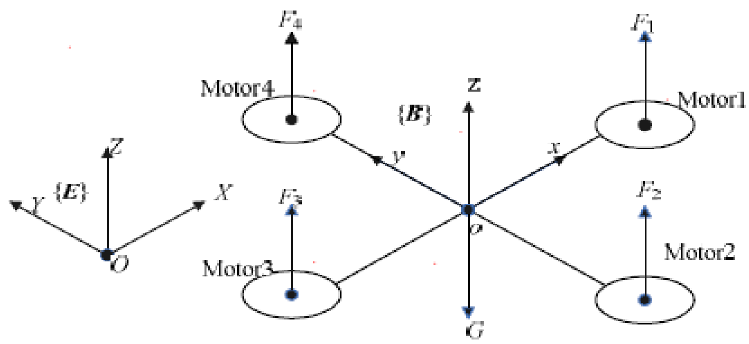

Figure 7 Coordinate system of four rotorcraft

(1) Earth's inertial reference coordinate system

Obviously, the inertial reference coordinate system of the earth is the earth coordinate system. So the absolute position of the aircraft is described by(x_0, y_0, z_0)three coordinates. The absolute position of the aircraft refers to the inertia reference coordinate system of the aircraft center of mass relative to the earth. The main function of this coordinate system is to describe the movement of the four rotorcraft relative to the ground. The origin coordinates are the same as the takeoff coordinates of the four rotorcraft. The X axis is the direction of the four rotorcraft taking off, the Z axis is vertically upward, the Y axis is perpendicular to the 0XZ plane and XYZ meets the left hand rule.

The attitude angle of the aircraft will be represented by three angles of $(\varphi, \theta, \emptyset)$. They are called yaw angle, pitch angle and roll angle. The range of the yaw angle$\varphi$is in the range of $(-\pi \leq \varphi < \pi)$; The pitch angle θ is in the range of $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$; The value of the roll angle $\emptyset$ is$-\frac{\pi}{2} < \emptyset < \frac{\pi}{2}$.

(2) aircraft body coordinate system

The coordinate system of the four rotorcraft is established based on the centroid and the center of gravity of the four rotorcraft. The Ox axis is the rotor of the four rotorcraft from the center of mass to the motor of the 1st motor. The Oy axis is perpendicular to the Ox axis and follows the robot arm of the four rotorcraft, from the center of the rotor of the four rotorcraft to the axle of the motor No. 2. The Oz axis is the axis perpendicular to the oxy plane and centered on the centroid.

(3) the conversion between the coordinate system

When we establish the inertial reference coordinate system and the aircraft body coordinate system of the earth's four rotorcraft, in order to describe the movement state of the four rotorcraft conveniently, it is necessary to master the conversion between the earth's inertial reference coordinate system and the aircraft body coordinate system relationship. First define three Euler angles, respectively, pitch angle θ, roll angle∅, yaw angle φ. The pitch angle θ refers to the angle at which the body of the four rotorcraft rotates around the yaxis with the x axis. The roll angle ∅ refers to the angle formed by the rotation of the four rotorcraft around the x-axis and the y-axis. Yaw angle φ refers to the four rotorcraft body around the z axis rotation angle.

In the initial state, the inertial reference coordinate system of the four rotorcraft coincides with the coordinate system of the four rotorcraft, and then rotates three times around the coordinate axis respectively to reach the new position. The rotation angles of the three times are the pitch angleθ , Roll angle∅, yaw angle φ. The three corners are transformed from the ground inertial reference coordinate system of the four rotorcraft to the transformation matrix of the body coordinate system of the four rotorcraft as follows.

The roll angle ∅ is formed by the rotation of the four-rotorcraft around the x-axis. The range of the roll angle ∅ is$\emptyset \in \left(-\frac{\pi}{2}, \ \frac{\pi}{2}\right)$

$$R_x(\emptyset) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\emptyset & sin\emptyset \\ 0 & -sin\emptyset & cos\emptyset \end{bmatrix} \quad (3\text{-}1)$$

The pitch angle θ is formed by rotating the four rotorcraft around the y axis, and the pitch angleθis in the range of valuesθ $\in \left(-\frac{\pi}{2}, \ \frac{\pi}{2}\right)$

$$R_y(\theta) = \begin{bmatrix} cos\theta & 0 & -sin\theta \\ 0 & 1 & 0 \\ sin\theta & 0 & cos\theta \end{bmatrix} \quad (3\text{-}2)$$

The formation of yaw angle φ is the rotation of the four rotorcraft around the z axis, the yaw angle φ of the value rangeφ $\in \left(-\frac{\pi}{2}, \ \frac{\pi}{2}\right)$

$$R_z(\varphi) = \begin{bmatrix} cos\varphi & sin\varphi & 0 \\ -sin\varphi & cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (3\text{-}3)$$

Therefore, in accordance with the yaw angle φ pitch angleθ roll angle ∅ in the order of rotation, you can get the four rotorcraft ground inertial reference coordinate system to the four rotorcraft body coordinate system transformation matrix:

$$R_{E-B} = R_x(\emptyset) \cdot R_y(\theta) \cdot R_z(\varphi) \quad (3\text{-}4)$$

That:

$$R_{E-B} = \begin{bmatrix} cos\theta cos\varphi & cos\theta sin\varphi & -sin\theta \\ sin\emptyset sin\theta cos\varphi - cos\theta sin\varphi & sin\emptyset sin\theta sin\varphi + cos\emptyset cos\varphi & sin\emptyset cos\theta \\ cos\emptyset sin\theta cos\varphi + sin\emptyset sin\varphi & cos\emptyset sin\theta sin\varphi - sin\emptyset cos\varphi & cos\emptyset cos\theta \end{bmatrix} \quad (3\text{-}5)$$

It can be easily converted from four rotorcraft body frame to the ground quadrotors inertial reference coordinate system, wherein the transformation matrix is as follows:

$$R_{B-E} = {R_{E-B}}^T = \begin{bmatrix} cos\theta cos\varphi & sin\emptyset sin\theta cos\varphi - cos\theta sin\varphi & cos\emptyset sin\theta cos\varphi + sin\emptyset sin\varphi \\ cos\theta sin\varphi & sin\emptyset sin\theta sin\varphi + cos\emptyset cos\varphi & cos\emptyset sin\theta sin\varphi - sin\emptyset cos\varphi \\ -sin\theta & sin\emptyset cos\theta & cos\emptyset cos\theta \end{bmatrix} \quad (3\text{-}6)$$

V. MATHEMATICAL MODEL OF THE ESTABLISHMENT

The movement of the four-rotorcraft under external forces can generally be described in the form of differential equations. The equations of motion of the four rotorcraft are divided into kinetic equations and kinematics equations[9]. The kinetic equation mainly describes the relationship between the force and torque on the rotor of the four rotorcraft and the movement of the four rotorcraft. And the kinematics equation is completely different from the dynamic equation, which does not need to consider those forces and moments acting on the four rotorcraft body, but the full use of geometrical theory and method to study the rigid body that is four rotorcraft movement.In this paper, the differential equations of the four rotorcraft are deduced from the angle of impulsive mechanics and kinematics, and the mathematical model of the four rotorcraft is simplified under the relevant constraint conditions.

For the four rotorcraft kinetic equation, according to the relevant literature review results show that the main use of two theoretical methods to derive.Using the Newton-Euler formula and the Euler-Lagrangian formula. In order to facilitate the research, this paper uses the easy-to-understand Newton-Euler formula to derive the kinetic equation of the four rotorcraft.

In order to facilitate the analysis and research of the four rotorcraft control algorithm and simplify the complexity of the problem, this paper makes the following assumptions on the dynamic model of the four rotorcraft:

(1) the four rotorcraft as rigid body, and the quality structure of uniform symmetry. The inertial matrix can be defined as diagonal matrix I.

(2) The center of gravity, center of mass and geometric center of the four rotorcraft are consistent with the origin of the two coordinate systems built for the aircraft.

(3) Ignoring the effects of air resistance and other surface factors on the four rotorcraft.

(4) The four elevators of the four rotorcraft are positively correlated with the square of the corresponding motor speed.

1. Line motion equation

Firstly, the whole force of the four-rotorcraft is analyzed according to the Newton's mechanics formula and the momentum theorem of motion. The main forces acting on the four-rotorcraft can be known, including the three parts, the lift force $F_T$produced by the rotor rotation of the four rotorcraft The gravity of the rotorcraft F_G, the air resistance of the four rotorcraftF_D.

The combined force acting to four rotorcraft quadrotors body frame reference coordinate system may be expressed as a combined force

$$\sum F^b = F_T + R_{E-B}F_G - F_D \quad (4\text{-}1)$$

In the body coordinate system, the four revolves of the four rotorcraft produce the total lift

$$F_r^b = [0 \quad 0 \quad \textstyle\sum_{i=1}^{4} F_i]^T \quad (4\text{-}2)$$

Where $F_i(i = 1,2,3,4)$ is the lift generated when each rotori rotates independently.

The gravityF_G^b is:

$$F_G^b = R_{E-B}F_G^e = R_{E-B}\begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} = \begin{bmatrix} mgsin\theta \\ -mgsin\emptyset cos\theta \\ -mgcos\emptyset cos\theta \end{bmatrix} \quad (4\text{-}3)$$

The air resistance of the four rotorcraft is:

$$F_D = K_d\dot{X}^e \quad (4\text{-}4)$$

Where K_d is a diagonal matrix and $K_d = diag(K_{dx}, K_{dy}, K_{dz})$is the rotation resistance coefficient matrix.

The above formula can be obtained:

$$\sum F^b = \begin{bmatrix} \mathrm{mgsin\theta} - K_{dz}\dot{X} \\ -mgsin\emptyset cos\theta - K_{dy}\dot{y} \\ \sum_{i=1}^{4} F_i - mgcos\emptyset cos\theta - K_{dz}\dot{z} \end{bmatrix} \quad (4\text{-}5)$$

Define $V^b = [u \ v \ w]^T$, V^b is the linear velocity vector of the body coordinate system of the four rotorcraft relative to the inertial coordinate system, where $\mathrm{u, v, w}$ are the linear velocity vector$V^b$of the four rotorcraft In the body coordinate system along thex_b, y_b, $z_b$axis linear velocity components.

The definition of $W^b = [p \ q \ r]^T$, $W^b$is the angular velocity vector of the body coordinate system of the four rotorcraft relative to the inertial coordinate system,where$\mathrm{p, q, r}$are the angular velocity vector$W^b$of the four rotorcraft respectively in the body The angular velocity component of the coordinate system along thex_b, y_b, z_b axis.

According to the momentum theorem of the general body motion, the kinetic equation of the centroid of the aircraft can be obtained as follows:

$$F^b = \mathrm{m}(\dot{V}^b + W^b \times V^b) \quad (4\text{-}6)$$

That:

$$\sum F^b = \begin{bmatrix} \mathrm{m}\left(\frac{du}{dt} + wq - vr\right) \\ m\left(\frac{dv}{dt} + ur - wp\right) \\ m\left(\frac{dw}{dt} + vp - uq\right) \end{bmatrix} \quad (4\text{-}7)$$

Therefore, combined with the above two formulas can be obtained in the body coordinate system under the three axial acceleration

$$\begin{cases} \dot{u} = \frac{1}{m}(mgsin\theta - K_{dz}\dot{x}) + vr - wq \\ \dot{v} = \frac{1}{m}\left(-mgsin\emptyset cos\theta - K_{dy}\dot{y}\right) + wp - ur \\ \dot{w} = \frac{1}{m}(\sum_{i=1}^{4} F_i - mgcos\emptyset cos\theta - K_{dz}\dot{z}) + uq - vp \end{cases} \quad (4\text{-}8)$$

According to the body coordinate system to the ground coordinate system of the change matrix can be analyzed by the four rotorcraft in the ground coordinate system, the joint force of the situation:

$$\sum F^e = R_{B-E}(F_T - F_D) + F_G^e \quad (4\text{-}9)$$

The thrust of the four rotorcraft is obtained by the coordinate transformation. The force under the ground coordinate system is expressed as:

$$F_r^e = R_{B-E}F_T^e = \sum_{i=1}^{4} F_i \begin{bmatrix} sin\theta cos\emptyset cos\varphi + sin\emptyset sin\varphi \\ sin\theta cos\emptyset sin\varphi - sin\emptyset cos\varphi \\ cos\emptyset cos\theta \end{bmatrix} \quad (4\text{-}10)$$

The gravity of the four rotorcraft can be expressed in the ground coordinate system as:

$$F_G^e = [0 \;\; 0 \;\; mg]^T \quad (4\text{-}11)$$

According to the above ground coordinate system under the formula, you can deduce:

$$\begin{cases} \ddot{x} = \frac{1}{m}[(sin\theta cos\emptyset cos\varphi + sin\emptyset sin\varphi)\sum_{i=1}^{4} T_i - K_{dx}\dot{x}] \\ y\ddot{} = \frac{1}{m}\left[(sin\theta cos\emptyset sin\varphi - sin\emptyset cos\varphi)\sum_{i=1}^{4} F_i - K_{dy}\dot{y}\right] \\ \ddot{z} = \frac{1}{m}[cos\emptyset cos\theta \sum_{i=1}^{4} F_i - K_{dz}\dot{z} - mg] \end{cases} \quad (4\text{-}12)$$

2. angular motion equation

Four rotorcraft in the role of torque, around the body of the center of mass to do the rotation angle movement.The four rotorcraft will be accompanied by the following effects of motion effects, aerodynamic effects, inertial anti-torque effects and gyro effects during flight. Due to the symmetrical structure features four rotorcraft, it can optionally quadrotor body frame origin centroid four rotorcraft, so that we can get the inertial FOURTH rotorcraft $I_{xy} = I_{yx} = I_{yz} = I_{zy} = I_{zx} = I_{xz} = 0$, about x, the moment of inertia y, z coordinate axis is not 0, the body of the inertia matrix I represents four rotorcraft as:

$$\mathrm{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \quad (4\text{-}13)$$

Where,I_x、I_y、I_z are the moment of inertia of the corresponding axis.

According to the Euler kinetic equation of the rigid body about the centroid rotation, the angular motion equation can be expressed as:

$$\sum M^b = I\dot{W}^b + W^b \times (IW^b) \quad (4\text{-}14)$$

In the formula, $W^b$is the angular velocity of the four rotorcraft in the body coordinate system relative to the four rotorcraft in the inertial coordinate system, I is the rotational inertia of the rotor of the four rotorcraft, and $M^b$is the rotational force of the four rotorcraft Acting on the body of the external torque.

In the above formula,$W^b \times (IW^b)$, can be written as follows:

$$W^b \times (IW^b) = \begin{bmatrix} i & j & k \\ p & q & r \\ I_x p & I_y q & I_z r \end{bmatrix} = \begin{bmatrix} rq(I_z - I_y) \\ pr(I_x - I_z) \\ pq(I_y - I_x) \end{bmatrix} \quad (4\text{-}15)$$

Combined with the above three formulas, into the finishing can get the following results:

$$\begin{cases} M_x = I_x\dot{p} + (I_z - I_y)qr \\ M_y = I_y\dot{q} + (I_x - I_z)pr \\ M_z = I_z\dot{r} + (I_y - I_x)pq \end{cases} \quad (4\text{-}16)$$

Among them, M_x, M_y, M_z are rigid body in x, y, z three coordinate axis direction of the moment component.

The body axis in three directions by the lift moment

$$\begin{bmatrix} M_{Tx} \\ M_{Ty} \\ M_{Tz} \end{bmatrix} = \begin{bmatrix} l(F_4 - F_2) \\ l(F_3 - F_1) \\ -M_{D1} + M_{D2} - M_{D3} + M_4 \end{bmatrix} \tag{4-17}$$

Where, l is the distance between the axis of rotation of the rotor center of mass four rotorcraft, $M_{Di}(i = 1,2,3,4$is four in the rotorcraft flight Quadrotor the z-axis by the moment, MDi=d*Ωi2*, where d is the drag coefficient of the rotor.

The four rotorcraft also produces a gyro effect during the actual flight. This is because the rotor of the four rotorcraft has a clockwise high speed rotation combination of counterclockwise high speed rotation group. When the four rotorcraft in the realization of pitching movement and roll movement, we must change the direction of the angular motion of the rotating aircraft, so as to generate torque. Then the gyro effect is generated in the rotor speed algebra and not equal to zero, will cause the body's imbalance, which will produce gyro torque, so that the four rotorcraft body rotation movement.

$$M_g = \sum_{i=1}^{4} W^b \times (I_r w_i) \tag{4-18}$$

Among them,$w_i = [0 \;\; 0 \;\; (-1)^{i+1}\Omega_i]^T$,$\Omega_i(i = 1,2,3,4)$ are the rotational speed of the left and right rotor, andI_r is the moment of inertia of the rotor.

The above formula can be simplified to give:

$$M_g = I_r \begin{bmatrix} -q \\ p \\ 0 \end{bmatrix} (-\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4) = \begin{bmatrix} -I_r q\Omega \\ I_r p\Omega \\ 0 \end{bmatrix} \tag{4-19}$$

Where $\Omega$is the algebraic sum of the four rotor rotations of the four rotorcraft, ie$\Omega = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4$. Obviously, due to the four rotorcraft, the four rotor in the rotation process of the gyro effect is only related to the angular velocity, and the line speed has nothing to do, so the above formula can be collated to get the four rotorcraft angular motion equation:

$$\begin{cases} I_x\dot{p} = (I_y - I_z)qr - I_r q\Omega + l(F_4 - F_2) \\ I_y\dot{q} = (I_y - I_x)pr - I_r p\Omega + l(F_3 - F_1) \\ \mathrm{I}_z\dot{r} = (I_x - I_y)pq + (-M_{D1} + M_{D2} - M_{D3} + M_{D4}) \end{cases} \tag{4-20}$$

In view of the complexity of the aerodynamics of the propeller of the four rotorcraft, this detailed and in-depth study is not allowed in time. Therefore, on the basis of predecessors, the aerodynamics of the propeller of the four rotorcraft The equation is simplified.

Assuming that the angular velocity of the rotor i of the four rotorcraft is $\Omega_i(i = 1,2,3,4)$, the lift of the single rotor of the four rotorcraft can be expressed as:

$$F_i = \mathrm{b}\Omega_i^2 \tag{4-21}$$

Where b is the lift coefficient of the rotor.
The above formula can be obtained:

$$F_{T合} = \sum_{i=1}^{4} F_i = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)$$

$$\begin{bmatrix} M_{Tx} \\ M_{Ty} \\ M_{Tz} \end{bmatrix} = \begin{bmatrix} l(F_4 - F_2) \\ l(F_3 - F_1) \\ -M_{D1} + M_{D2} - M_{D3} + M_{D4} \end{bmatrix} = \begin{bmatrix} lb(\Omega_4^2 - \Omega_2^2) \\ lb(\Omega_3^2 - \Omega_1^2) \\ d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \end{bmatrix} \tag{4-22}$$

Based on the above analysis and research, the nonlinear motion equation of the four rotorcraft can be expressed as follows:

$$\begin{cases} \begin{cases} \ddot{x} = \frac{1}{m}\left[(sin\theta cos\emptyset cos\varphi + sin\emptyset sin\varphi)\sum_{i=1}^{4} T_i - D_x\right] \\ \ddot{y} = \frac{1}{m}\left[(sin\theta cos\emptyset sin\varphi - sin\emptyset cos\varphi)\sum_{i=1}^{4} T_i - D_y\right] \\ \ddot{z} = \frac{1}{m}\left[cos\emptyset cos\theta \sum_{i=1}^{4} T_i - D_z - mg\right] \end{cases} \\ \begin{cases} \ddot{\emptyset} = \frac{1}{I_x}\left[l(T_4 - T_2) - (I_z - I_y)\dot{\theta}\dot{\varphi}\right] \\ \ddot{\theta} = \frac{1}{I_y}\left[l(T_3 - T_1) - (I_x - I_z)\dot{\emptyset}\dot{\varphi}\right] \\ \ddot{\varphi} = \frac{I}{I_z}\left[(M_1 - M_2 + M_3 - M_4) - (I_y - I_x)\dot{\emptyset}\dot{\theta}\right] \end{cases} \end{cases} \quad (4\text{-}23)$$

VI. MODEL SIMPLIFICATION

From the above formula, it can be clearly found that the nonlinear model of the four rotorcraft is composed of two parts: the line motion and the angular motion, and the angular motion affects the movement of the line between the angular motion and the line motion. In order to facilitate the research carried out, this paper accordingly ignores the four rotorcraft in the work of the external complex environment. This is equivalent to the state where the four rotorcraft is in a windless condition or at a low speed flight or hover. So there are:

$$\dot{\emptyset}\dot{\varphi} \approx 0,\ \dot{\theta}\dot{\varphi} \approx 0,\ \dot{\emptyset}\dot{\theta} \approx 0 \quad (5\text{-}1)$$

In order to simplify the problem and describe the convenience, the dynamic subsystem is separated from the whole dynamic model. In the study, the four rotor speeds of the four rotorcraft are regarded as control inputs, which decompose the nonlinear model of the four rotorcraft into four Independent control, the system of virtual control input is defined as:

$$\begin{cases} U_1 = \sum_{i=1}^{4} F_i = b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ U_2 = l(F_4 - F_2) = lb(\Omega_4^2 - \Omega_2^2) \\ U_3 = l(F_3 - F_1) = lb(\Omega_3^2 - \Omega_1^2) \\ U_4 = -M_{D1} + M_{D2} - M_{D3} + M_{D4} = d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2) \end{cases} \quad (5\text{-}2)$$

The virtual control input of the system is brought into the nonlinear equation of the system of the four rotorcraft, and a simplified mathematical model of the four rotorcraft represented by the position and attitude angle is obtained.

$$\begin{cases} \ddot{x} = (sin\theta cos\emptyset cos\varphi + sin\emptyset sin\varphi)\frac{U_1}{m} \\ \ddot{y} = (sin\theta cos\emptyset sin\varphi - sin\emptyset cos\varphi)\frac{U_1}{m} \\ \ddot{z} = cos\emptyset cos\theta \frac{U_1}{m} - g \\ \ddot{\emptyset} = \frac{(I_y - I_z)}{I_x}\dot{\theta}\dot{\varphi} + \frac{U_2}{I_x} \\ \ddot{\theta} = \frac{(I_z - I_x)}{I_y}\dot{\emptyset}\dot{\varphi} + \frac{U_3}{I_y} \\ \ddot{\varphi} = \frac{(I_x - I_y)}{I_z}\dot{\emptyset}\dot{\theta} + \frac{U_4}{I_z} \end{cases} \quad (5\text{-}3)$$

That:

$$\begin{cases} \ddot{x} = (sin\theta cos\emptyset cos\varphi + sin\emptyset sin\varphi)\frac{U_1}{m} \\ \ddot{y} = (sin\theta cos\emptyset sin\varphi - sin\emptyset cos\varphi)\frac{U_1}{m} \\ \ddot{z} = cos\emptyset cos\theta \frac{U_1}{m} - g \\ \ddot{\emptyset} = \frac{U_2}{I_x} \\ \ddot{\theta} = \frac{U_3}{I_y} \\ \ddot{\varphi} = \frac{U_4}{I_z} \end{cases} \quad (5\text{-}4)$$

VII. CONCLUSION

This chapter introduces the mechanical structure and characteristics of the four rotorcraft and the principle of motion. Two coordinate systems are defined for the motion characteristics of the four rotorcraft, and the forces and moments of the four rotorcraft are analyzed on the basis of two coordinate systems. The nonlinear model of the four rotorcraft can be obtained by deriving the dynamic model of the four rotorcraft by the Newton-Euler formula. On the basis of the reasonable assumption, the four rotorcraft Of the simplified mathematical model for the following types of algorithms to provide an important theoretical basis.

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